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CP violation due to multi-Froggatt–Nielsen fields

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Abstract. We study how to incorporate *CP* violation in the Froggatt–Nielsen (FN) mechanism. To this end, we introduce non-renormalizable interactions with a flavor democratic structure to the fermion mass generation sector. It is found that at least two iso-singlet scalar fields with a discrete symmetry imposed are necessary to generate *CP* violation due to the appearance of the relative phase between their vacuum expectation values. In the simplest model, the ratios of quark masses and the Cabibbo–Kobayashi–Maskawa (CKM) matrix including the *CP* violating phase are determined by the CKM element $|V_{us}|$ and the ratio of two vacuum expectation values of FN fields, $R = |R|e^{i\alpha}$ (a magnitude and a phase). It is demonstrated how the angles ϕ_i $(i = 1, \dots, 3)$ of the unitarity triangle and the CKM off-diagonal elements $|V_{ub}|$ and $|V_{cb}|$ are predicted as a function of $|V_{us}|$, $|R|$ and α. Although the predicted value of the *CP* violating phase does not agree with the experimental data within the simplest model, the basic idea of our scenario would be promising if one wants to construct a more realistic model of flavor and *CP* violation.

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1 Introduction

The standard model (SM) of electroweak interactions has been successful. It can explain all experimental results except for neutrino oscillation phenomena. Masses of quarks and leptons are generated through the Yukawa interaction after electroweak symmetry breaking. However, no principle has been established to determine the flavor structure of the Yukawa couplings, and the origin of the fermion mass hierarchy remains unknown.

There have been many attempts to explain the flavor structure of the Yukawa couplings. A promising approach would be the idea of flavor symmetry. In models based on the Froggatt–Nielsen (FN) mechanism [1], the U(1) global symmetry is imposed as a flavor symmetry, in which the vacuum expectation value (VEV) of an iso-singlet scalar field (FN field) gives a power-like structure of the Yukawa couplings due to the U(1) charge assignment for the relevant fields. The extension to more complicated flavor symmetries has also been studied; i.e., non-Abelian global symmetries such as U(2) [2–7], discrete symmetries such as S_3 [8–13], A_4 [14–16], D_5 [17], etc. These have several distinct patterns of symmetry breaking, and the difference in VEVs for each scalar field gives a hierarchical structure to the Yukawa matrix. In most of such models, only the orders of magnitude of the Yukawa matrix elements are estimated, so that $\mathcal{O}(1)$ uncertainties exist in the coefficients of the coupling constants between the scalar and matter fields. In this framework, *CP* violation (CPV) comes from complex phases in these coefficients.

On the other hand, some kinds of texture such as a democratic structure [18–24] have been investigated for the Yukawa matrix. In the model with the democratic structure, all the elements of the Yukawa matrix are assumed to have the same value up to leading order, and the mass hierarchy and flavor mixing are given by diagonalizing these Yukawa matrices. CPV is supposed to appear as a consequence of the complex nature of the terms of tiny breaking of the democratic structure. In any case the *CP* violating phase comes from complex phases of free parameters, so that it is not predictable.

Since both flavor mixing and *CP* violating phase are determined by the Yukawa matrices, it would be natural to consider them to be given by the same mechanism as is relevant to the Yukawa interaction. In the scenario of spontaneous CPV [25], the phase is deduced from the relative complex phase between the VEVs of the scalar

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fields. Recently, this idea is applied for an SO(10) model with complex VEVs of the Higgs field [26, 27]. Combining the spontaneous CPV scenario with the idea of the flavor symmetry, one can obtain the non-zero complex phase in the Yukawa matrix from the VEVs of the scalar fields. This idea has been developed in several flavor models; e.g., a model with three U(1) flavon scalar fields [28], models with non-Abelian flavor symmetry [29–31], etc.

In this paper, we introduce a simple model in which the FN mechanism works with a democratic Yukawa structure between quarks and FN fields, and we show how the CPV can be obtained. As mentioned in [28], this type of models requires at least more than two FN fields for a successful prediction of the physical *CP* violating phase.

This paper is organized as follows. In Sect. 2, we study the generation of CPV for quarks based on the FN mechanism with the democratic ansatz. In Sect. 3, simple models with two FN fields are discussed. We present analytic expressions for the CKM parameters, and numerical evaluations are also shown. Conclusions are given in Sect. 4.

2 CP violation in democratic models

The democratic ansatz for the flavor structure of the Yukawa matrix has been implemented in [18–24, 34–42]. In this framework, the Yukawa matrices for the up- and down-type quarks are simply written as

$$Y_{u,d} \propto \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} . \tag{1}$$

This flavor structure can be constructed by models with the $S_{3R} \times S_{3L}$ permutation symmetry [34–37]. The symmetry can be realized in the geometrical origin of the brane-world scenario [38, 39], and also in the strong dynamics [40–42]. Two of the three eigenvalues are zero in these matrices in (1), and no *CP* violating phase appears in the $S_{3R} \times S_{3L}$ limit[1]. It is clear that in order to explain the experimental data this permutation symmetry must be broken by some small effects. When the small breaking terms for the permutation and *CP* symmetries are introduced by hand, the mass splitting between the first and second generation quarks, the mixing angles and the *CP* violating phase are explained.

The FN mechanism is a simple idea to generate the mass hierarchy of quarks and leptons. In the simplest FN model [1], an iso-singlet scalar field, Θ, is introduced with the $U(1)_{FN}$ flavor symmetry in order to discriminate the fermion flavor by the $U(1)_{FN}$ charge.

The $U(1)_{FN}$ charge assigned to Θ is taken to be $f_\Theta = -1$ without loss of generality. Under the $U(1)_{FN}$ symmetry, non-renormalizable interactions relevant to the quark mass matrix can be written as[2]

$$\mathcal{L}_{FN} = -(C_U)_{ij} \bar{U}_i Q_j \cdot H_u \left(\frac{\Theta}{\Lambda}\right)^{f_{U_i^c}+f_{Q_j}+f_{H_u}} - (C_D)_{ij} \bar{D}_i Q_j \cdot H_d \left(\frac{\Theta}{\Lambda}\right)^{f_{D_i^c}+f_{Q_j}+f_{H_d}} , \tag{3}$$

where H_u and H_d are iso-doublet fields (the Higgs fields) with their hypercharge being $-1/2$ and $+1/2$, respectively[3], Q_i is the left-handed quark doublet, U_i and D_i are right-handed up- and down-type quarks in the ith generation, and C_U and C_D are coupling constants of order one. The $U(1)_{FN}$ charge of the field X is expressed by f_X. The cutoff scale is given by Λ, which describes the mass scale of the new physics dynamics. The coefficients $(C_U)_{ij}$ and $(C_D)_{ij}$ are generally complex numbers.

After $U(1)_{FN}$ is broken by the VEV of Θ,

$$\langle\Theta\rangle = \lambda\Lambda , \tag{4}$$

where λ is a small dimensionless parameter, the quark Yukawa matrices are obtained as

$$(Y_{U,D})_{ij} = (C_{U,D})_{ij} \lambda^{f_{U_i^c, D_i^c}+f_{Q_j}+f_{H_{u,d}}} . \tag{5}$$

With the assignment of the $U(1)_{FN}$ charges (see e.g. [43–46]) as follows:

$$\begin{aligned} ((f_{Q_1}, f_{U_1^c}), (f_{Q_2}, f_{U_2^c}), (f_{Q_3}, f_{U_3^c})) &= (3, 2, 0) , \\ (f_{D_1^c}, f_{D_2^c}, f_{D_3^c}) &= (2, 1, 1) , \\ (f_{H_u}, f_{H_d}) &= (0, 0) , \end{aligned} \tag{6}$$

the observed quark mass hierarchy and the CKM mixings can be derived by assuming λ to be close to the Cabibbo angle $\sin\theta_c = 0.22$. At leading order, the induced masses for the quarks are $m_u \sim \lambda^6\langle H_u\rangle$, $m_c \sim \lambda^4\langle H_u\rangle$, $m_t \sim \langle H_u\rangle$, $m_d \sim \lambda^5\langle H_d\rangle$, $m_s \sim \lambda^3\langle H_d\rangle$ and $m_b \sim \lambda\langle H_d\rangle$, and the CKM matrix is given by

$$U_{CKM} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} . \tag{7}$$

Now we consider the possibility of spontaneous CPV due to the complex phase of $\langle\Theta\rangle$. We assume that C_U and C_D in (3) have the democratic structure, i.e.,

$$C_{U,D} = \alpha_{U,D} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} . \tag{8}$$

[1] Keeping the $S_{3R} \times S_{3L}$ symmetry, the complex Yukawa matrices $Y_{u,d}$ in (1) can be re-expressed, for example, by an appropriate unitary transformation as follows:

$$Y_{u,d} \propto \begin{pmatrix} \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} , \tag{2}$$

where $\omega = e^{i2\pi/3}$ is the cube root of one. However, the complex phase in this matrix is unphysical, because it is rephased out by the redefinition of the quark fields.

[2] There are possibilities to consider additional terms including powers of $(\Theta_i\Theta_j^*/\Lambda^2)$. Yet we neglect them at leading order, since these interactions are always suppressed by λ^{2n}.

[3] In the SM, $H_d = i\sigma_2 H_u^*$ is satisfied.

There is no CPV in the model with only one FN field. Although complex phases may be obtained in the mass matrices by introducing the complex VEV of Θ, such phases are rotated away by the phase redefinition of the quark fields. Hence the model should have at least two FN fields $\Theta_{1,2}$ in order to obtain a non-vanishing *CP* violating phase. We start from the following Lagrangian with two FN fields:

$$\mathcal{L}_{\mathrm{FN}_2} = -\sum_{n_1^u, n_2^u} \bar{U}_i (C_U)_{ij} Q_j \cdot H_u \left(\frac{\Theta_1}{\Lambda}\right)^{n_1^u} \left(\frac{\Theta_2}{\Lambda}\right)^{n_2^u} - \sum_{n_1^d, n_2^d} \bar{D}_i (C_D)_{ij} Q_j \cdot H_d \left(\frac{\Theta_1}{\Lambda}\right)^{n_1^d} \left(\frac{\Theta_2}{\Lambda}\right)^{n_2^d}, \quad (9)$$

where $n_1^{u,d}$ and $n_2^{u,d}$ run from zero to $n_{ij}^{U,D} \equiv f_{U_i^c, D_i^c} + f_{Q_j} + f_{H_{u,d}}$, satisfying the constraint $n_1^{u,d} + n_2^{u,d} = n_{ij}^{U,D}$. After the $\mathrm{U(1)_{FN}}$ symmetry is broken, the Yukawa couplings in the SM are given by

$$(Y_{U,D})_{ij} = (C_{U,D})_{ij} \lambda^{n_{ij}^{U,D}} \sum_{k=0}^{n_{ij}^{U,D}} R^k, \quad (10)$$

where

$$\lambda = \frac{\langle\Theta_1\rangle}{\Lambda}, \quad R = \frac{\langle\Theta_2\rangle}{\langle\Theta_1\rangle} \equiv |R| \mathrm{e}^{\mathrm{i}\alpha}. \quad (11)$$

Therefore, the physical *CP* violating phase can be obtained from the relative phase α between $\langle\Theta_1\rangle$ and $\langle\Theta_2\rangle$.

3 Examples for the model with two Froggatt–Nielsen fields

In this section, we show how to generate CPV from two FN fields by considering simple models. In order to generate the quark mass hierarchy, we employ the $\mathrm{U(1)_{FN}}$ charge assignment for matter fields given in (6). In general, $\mathrm{U(1)_{FN}}$ charges for the FN fields Θ_1 and Θ_2 can be different from each other. We here assume that they both have the same $\mathrm{U(1)_{FN}}$ charge for simplicity: $(f_{\Theta_1}, f_{\Theta_2}) = (-1, -1)$.

The simplest toy model

First of all, we discuss the naive model defined in (9). The mass matrices $M_{u,d}$ for up- and down-type quarks are given by

$$(M_u)_{ij} = \alpha_U \langle H_u\rangle A_{n_{ij}^U} \lambda^{n_{ij}^U}, \quad (M_d)_{ij} = \alpha_D \langle H_d\rangle A_{n_{ij}^D} \lambda^{n_{ij}^D}, \quad (12)$$

where $A_n = \sum_{k=0}^n R^k$. This model predicts $m_s^2/m_b^2 = \mathcal{O}(\lambda^6)$ and $|V_{us}| = \mathcal{O}(\lambda)$. Only one of the two experimental values can be adjusted. Furthermore, each mass matrix gives one massless eigenstate because of the conditions $\det M_u = \det M_d = 0$[4].

In order to avoid these difficulties, the following possibilities can be considered: introducing an additional symmetry, or throwing away the democratic ansatz given in (8), etc. In the next model, we explore the possibility of keeping the democratic structure for $C_{U,D}$.

Extended models with Z_2 symmetry

We try to construct a more realistic model. The $\mathrm{U(1)_{FN}}$ charges are assigned again as in (6). In order to obtain the observed value of m_s^2/m_b^2 by setting $\lambda \sim \sin\theta_\mathrm{c}$, we impose the Z_2 symmetry under the transformation of $\Theta_1 \to \Theta_1$ and $\Theta_2 \to -\Theta_2$. For the Z_2 parity assignment for the quark fields, there are a lot of choices. If we consider the scenario associated with the grand unified theory (GUT), it would be natural that the Z_2 parity for Q_i and U_i^c is common and that for H_u is set to be + for the prediction of a large top-quark mass. Because H_d always couples to D_i^c, we can set the Z_2 parity for H_d to be + without loss of generality. Then, there are 64 possibilities on parity assignment for the quarks. However, it turns out that most of them cannot give correct values of m_c^2/m_t^2 and m_s^2/m_b^2. Consequently, only the following sets of Z_2 parity assignment suffice for our discussion.

For type I-a, we have

$$\left((Q_1, U_1^c), (Q_2, U_2^c), (Q_3, U_3^c)\right) = (+,+,-), \quad \left(D_1^c, D_2^c, D_3^c\right) = (+,+,-). \quad (13)$$

For type I-b, we have

$$\left((Q_1, U_1^c), (Q_2, U_2^c), (Q_3, U_3^c)\right) = (+,+,-), \quad \left(D_1^c, D_2^c, D_3^c\right) = (-,+,-). \quad (14)$$

For type II-a, we have

$$\left((Q_1, U_1^c), (Q_2, U_2^c), (Q_3, U_3^c)\right) = (+,-,+), \quad \left(D_1^c, D_2^c, D_3^c\right) = (+,-,+). \quad (15)$$

For type II-b, we have

$$\left((Q_1, U_1^c), (Q_2, U_2^c), (Q_3, U_3^c)\right) = (+,-,+), \quad \left(D_1^c, D_2^c, D_3^c\right) = (-,-,+). \quad (16)$$

Type I-a gives the mass matrices as

$$M_u(\text{I-a}) = \begin{pmatrix} B_6\lambda^6 & B_4\lambda^5 & RB_2\lambda^3 \\ B_4\lambda^5 & B_4\lambda^4 & R\lambda^2 \\ RB_2\lambda^3 & R\lambda^2 & 1 \end{pmatrix} \alpha_U \langle H_u\rangle, \quad M_d(\text{I-a}) = \begin{pmatrix} B_4\lambda^5 & B_4\lambda^4 & R\lambda^2 \\ B_4\lambda^4 & B_2\lambda^3 & R\lambda \\ RB_2\lambda^4 & RB_2\lambda^3 & \lambda \end{pmatrix} \alpha_D \langle H_d\rangle, \quad (17)$$

where $B_{2n} = \sum_{k=0}^n R^{2k}$. Diagonalizing the above matrices, we obtain the mass ratios m_c^2/m_t^2, m_s^2/m_b^2, the CKM mix-

[4] Even when the u- and d-quarks are massless at the electroweak scale, their finite masses would be generated at lower energy scales due to the strong dynamics [47, 48].

ing angles (absolute values of CKM matrix elements), and the Kobayashi–Maskawa phase $\phi_3 \equiv \arg(V_{ub}^* V_{ud}/V_{cb}^* V_{cd})$ at the leading order:

$$\frac{m_c^2}{m_t^2} = |1+R^4|^2\lambda^8 \,, \qquad \frac{m_s^2}{m_b^2} = \frac{|1-R^4|^2}{(1+|R|^2)^2}\lambda^4 \,,$$
$$|V_{us}| = \frac{2}{\sqrt{|R|^8+|R|^{-8}-2(2\cos^2 4\alpha -1)}}\lambda \,,$$
$$|V_{ub}| = \frac{|R|\,\big||R|-|R|^{-1}\big|}{(|R|+|R|^{-1})\sqrt{|R|^4+|R|^{-4}+2\cos 4\alpha}}\lambda^3 \,,$$
$$|V_{cb}| = \frac{|R|\sqrt{|R|^2+|R|^{-2}+2\cos 4\alpha}}{|R|+|R|^{-1}}\lambda^2 \,,$$
$$\phi_1 = \arg\left\{ |R|^4 + \frac{1}{|R|^2} + (|R|^2+1)\cos 4\alpha - \mathrm{i}(|R|^2-1)\sin 4\alpha \right\} ,$$
$$\phi_2 = \arg\left\{ (1-|R|^2)\left(\frac{1}{|R|^4} - |R|^4 + 2\mathrm{i}\sin 4\alpha\right)\right\} ,$$
$$\phi_3 = \arg\left\{ (1-|R|^2)\left(|R|^4 - \frac{1}{|R|^2} + (|R|^2-1)\cos 4\alpha + \mathrm{i}(|R|^2+1)\sin 4\alpha \right)\right\} . \qquad (18)$$

Let us discuss the appropriate values of $|R|$, $\cos 4\alpha$ and $\sin 4\alpha$. We first expect that $\lambda \sim \sin\theta_c$. Then, $|R|>1$ is needed to obtain a reasonable value for $|V_{ub}|$. However, $|R|$ cannot be much larger than unity, because m_c^2/m_t^2 exceeds the experimentally acceptable value. In addition, $\cos 4\alpha < 0$ is necessary for $|R|>1$ to explain the data for $|V_{cb}|$. Finally, $\sin 4\alpha < 0$ is required for ϕ_1 to be in the first quadrant. In this case, however, it turns out that ϕ_3 cannot be in the first quadrant simultaneously.

For numerical evaluation, we take $|R| = \sqrt{3/2}$, $\cos 4\alpha = -3/4$ and $\sin 4\alpha = -\sqrt{7}/4$. This parameter set determines $\lambda = 0.25$ under the experimental value of $|V_{us}| = 0.22$. The matrices $M_{u,d}$(I-a) are diagonalized, and we obtain

$$|V_{ub}| = 0.0028 \,, \quad |V_{cb}| = 0.032 \,, \qquad (19)$$

which values are in excellent agreement with the CKM mixing angles at the GUT scale, $|V_{cb}| = 0.029$–0.039 and $|V_{ub}| = 0.0024$–0.0038, which are evaluated by renormalization group methods with the experimental data at low energies [49, 50]. However, the quark mass ratios are predicted to be

$$\frac{m_c}{m_t} = 0.0061 \,, \quad \frac{m_s}{m_b} = 0.073 \,, \qquad (20)$$

which are about twice as large as the expected values at the GUT scale: $m_c/m_t \sim 0.0032 \pm 0.0007$ and $m_s/m_b = 0.036 \pm 0.005$. Finally, we find

$$\phi_1 = 12^\circ \,, \quad \phi_2 = 31^\circ \,, \quad \phi_3 = 137^\circ \,. \qquad (21)$$

Although the predictions cannot explain all the data simultaneously, it would be amazing if one were to observe that this model can reproduce most of them to a considerable extent. We also find that for type I-b, the mixing angles, the mass ratios between the second and third generations, and the CKM phase are completely the same at leading order as in (18).

For type II-a, the mass matrices are

$$M_u(\text{II-a}) = \begin{pmatrix} B_6\lambda^6 & RB_4\lambda^5 & B_2\lambda^3 \\ RB_4\lambda^5 & B_4\lambda^4 & R\lambda^2 \\ B_2\lambda^3 & R\lambda^2 & 1 \end{pmatrix} \alpha_U \langle H_u \rangle \,,$$
$$M_d(\text{II-a}) = \begin{pmatrix} B_4\lambda^5 & RB_2\lambda^4 & B_2\lambda^2 \\ RB_2\lambda^4 & B_2\lambda^3 & R\lambda \\ B_4\lambda^4 & RB_2\lambda^3 & \lambda \end{pmatrix} \alpha_D \langle H_d \rangle \,. \qquad (22)$$

The resulting mass ratios, the CKM parameters and the phases ϕ_1, ϕ_2 and ϕ_3 are the same as those in type I, except for $|V_{us}|$ and $|V_{ub}|$, which are

$$|V_{us}| = \frac{2|R|}{\sqrt{|R|^8+|R|^{-8}-2(2\cos^2 4\alpha -1)}}\lambda \,,$$
$$|V_{ub}| = \frac{|R|^2||R|-|R|^{-1}|}{(|R|+|R|^{-1})\sqrt{|R|^4+|R|^{-4}+2\cos 4\alpha}}\lambda^3 \,. \qquad (23)$$

These expressions are different from those in type I by the multiplication factor $|R|$.

In this case, we take $|R| = \sqrt{3/2}$ and $\cos 4\alpha = -1/2$ in order to compensate the effect of the extra factor $|R|$ in $|V_{us}|$ in comparison with that in type I. In addition, we take $\sin 4\alpha = -\sqrt{3}/2$ and $\lambda = 0.23$. We obtain the following results:

$$|V_{us}| = 0.22 \,, \quad |V_{ub}| = 0.0038 \,, \quad |V_{cb}| = 0.035 \,,$$
$$\frac{m_c}{m_t} = 0.0060 \,, \quad \frac{m_s}{m_b} = 0.059 \,,$$
$$\phi_1 = 18^\circ \,, \quad \phi_2 = 38^\circ \,, \quad \phi_3 = 123^\circ \,. \qquad (24)$$

Although the size of m_s/m_b becomes smaller than that of type I, it is still too large to be phenomenologically acceptable. Moreover, m_c/m_t remains two times greater than the expected value, and ϕ_3 is in the second quadrant. Type II-b gives almost the same results for the CKM parameters and the mass ratios between the second and third generations.

4 Conclusion

We have studied the possibility of incorporating CPV by using the FN mechanism in the context of democratic flavor FN couplings. We have considered models with two FN fields, in which the relative phase of their VEVs plays the role of the origin of the *CP* violating phase at low energies. In the scenario with Z_2 symmetry, the relationship among the ratios of the quark masses, the absolute values of the CKM matrix elements and the *CP* violating phase has been examined in several of the simplest models. We have found that the predictions have been in good agreement with most of the data. However, the *CP* violating phase ϕ_3 has been predicted to be around 130°, so that the models we have examined are not acceptable. It may be an oversimplification to assume flavor blind couplings. We expect that a small modification for the democratic assumption would cure this phenomenological problem.

We have demonstrated the way how to introduce the CPV to the FN model and have shown that the scenario with two FN fields would be promising. An application of our scenario to the lepton sector including neutrinos is under way and will appear in our future publications. If this will be successfully achieved, we would obtain a model that gives a unified description of the *CP* phases for quarks and leptons.

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